

AP Calculus AB

Volume of Solids (Cross Section)

1) $f(x) = x \sin x$

$$f_{avg} = \frac{1}{\pi-1} \int_1^{\pi} f(x) dx$$

$$= 1.326$$

2) $y_1 = f(x)$ $y_2 = g(x)$

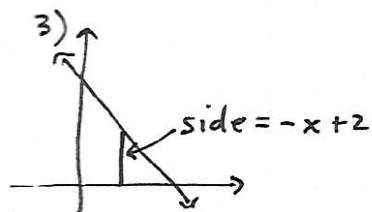
$$y_1 = y_2 @ x=0$$

$$x = 1.032 \rightarrow B$$

$$x = 2$$

$$\text{Area} = \int_0^B [y_2 - y_1] dx + \int_B^2 [y_1 - y_2] dx$$

$$= 2.004$$



$$\text{Volume} = \int \text{Area of a Square}$$

$$= \int \text{side}^2 dx$$

$$V = \int_0^2 (-x+2)^2 dx$$

$$= \int_0^2 (x^2 - 4x + 4) dx$$

$$\left. \frac{1}{3}x^3 - 2x^2 + 4x \right|_0^2$$

$$\frac{8}{3} - 8 + 8 = \boxed{\frac{8}{3}}$$

5) $y_1 = e^{-x^2}$ $y_2 = 1 - \cos x$

$$y_1 = y_2 @ x = 0.941 \rightarrow B$$

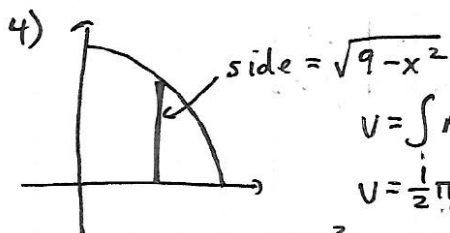
$$\text{Volume} = \int \text{Area of a Square}$$

$$= \int \text{side}^2$$

$$\text{side} = y_1 - y_2$$

$$V = \int_0^B [y_1 - y_2]^2 dx$$

$$= 0.461$$



$$V = \int \text{Area of a Semicircle}$$

$$V = \frac{1}{2} \pi \int \left(\frac{\text{side}}{2} \right)^2$$

$$V = \frac{\pi}{8} \int_0^3 (9 - x^2) dx$$

$$\left. \frac{\pi}{8} \left(9x - \frac{1}{3}x^3 + C \right) \right|_0^3$$

$$\frac{\pi}{8} (27 - 9) = \boxed{\frac{18}{8}\pi}$$

6) $y_1 = \sqrt{x}$ $y_2 = e^{-3x}$

$$y_1 = y_2 @ x = 0.238 \rightarrow A$$

$$V = \int \text{Area of a Rectangle}$$

$$= \int \text{base} \cdot \text{height}$$

$$\text{base} = y_1 - y_2 \quad \text{height} = 5 \cdot \text{base} \\ 5(y_1 - y_2)$$

$$V = \int_A^1 (y_1 - y_2) \cdot 5(y_1 - y_2) dx$$

$$= 5 \int_A^1 (y_1 - y_2)^2 dx$$

$$= 1.554$$

$$7) \int \frac{x}{\sqrt{9-x^2}} dx$$

$$\int x(9-x^2)^{-1/2} dx$$

$$\boxed{-(9-x^2)^{1/2} + C}$$

check

$$\frac{1}{2}(9-x^2)^{-1/2} \cdot -2x$$

$$8) f'(\frac{\pi}{3}) = 0.457$$

$$9) y = \frac{2x^2}{4-x^2} = \frac{2x^2}{(2-x)(2+x)}$$

remov. disc @ $x=2$ & $x=-2$

TWO VERTICAL ASYMP.

$$\lim_{x \rightarrow \infty} \frac{2x^2}{4-x^2} = -2$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2}{4-x^2} = -2$$

HORIZ ASYMP @ $y=-2$

(E)

$$10) A(t) = 6.687(0.931)^t$$

$$a) ROC = \frac{A(30) - A(0)}{30 - 0} = -0.1968 \frac{\text{pounds}}{\text{day}}$$

$$b) A'(15) = -0.1636 \text{ lbs/day}$$

On day 15, glass clippings remaining in the bin are decreasing at a rate of 0.1636 lbs/day.